ture; p_i, initial pressure; x, coordinate; A and D, auxiliary quantities, determined by the relations (10) and (12); r, temperature restoration coefficient; ξ, relative mass fraction of the gas phase; C_1, a constant; Re, Reynolds number; Pr, Prandtl's number; μ_a, dynamic viscosity of the gas; d_b, r_b, diameter and radius of the particles, respectively; C_D, coefficient of resistance of a spherical particle; Nu, Nusselt number; α and β, coefficients of proportionality of the velocity; m_i, ratio of the mass flow rates of the liquid (solid) and gas phases; k, ratio of the velocities of the liquid (solid) and gaseous phases; $a = (1 - \xi)/\rho_b$; p_0, pressure for $a = 0$; p_1, a small pressure perturbation for $a \neq 0$; m and δ, quantities determined by the formulas (23) and (26); y, an auxiliary function, determined by the expression (30); l, length of the nozzle. Indices: a, gas phase; b, liquid (solid) phase; i, initial state; and f, final state.

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FLOW STABILITY OF A FILM OF VISCOUS LIQUID ON THE SURFACE OF A ROTATING DISK

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UDC 532.516

The stability of a steady axisymmetric flow of a film is studied using the assumption of local plane parallelity. We present the results of numerical calculation.

The flow of a film along the surface of a flat rotating disk is encountered in many technological processes. An example is the preparation of metal powder by the centrifugal method. In the present work we study the linear stability of a steady axisymmetric flow of a film with a relatively small thickness.

Let us suppose that a viscous incompressible liquid is supplied at a constant flow rate near the center of a rotating disk. To describe the flow of the film which is formed on the disk we choose the functions [1]

$$u = \frac{u_r}{\Omega r \delta^2}, \quad v = \frac{1}{\delta^2}\left(\frac{u_\theta}{\Omega r} - 1\right), \quad w = \frac{u_z}{\Omega H_c \delta^2}, \quad p = \frac{p_f}{\rho \Omega^2 H_c^2},$$

where $\delta = H_c\sqrt{\Omega/\nu}$. The independent variables are chosen as $x = \ln(r/R)$, θ, $y = z/H_c$, $s = \Omega t/\delta^2$.

The functions u, v, w, and p, which depend on x, θ, y, and s and on the form of the free surface of the film $h(x, \theta, s)$, are determined by solving the system of Navier–Stokes equations, the sticking and impermeability conditions on the surface of the disk, the kinematic condition on the free surface where we have also the conditions of zero tangential stress along two directions, and the condition that the stress normal to the surface in the liquid is equal to the stress of capillary forces [1].

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The problem of steady axisymmetric flow of the film with a relatively small thickness has an asymptotic solution [1]. Neglecting the surface tension and terms of order ε^2, where $\varepsilon(x) = H_C \exp(-x)/R$, the principal terms in the expansion of the radial and azimuthal velocity components of this solution in a series in δ^4 have the form

$$U_1 = H_1 y - \frac{1}{2} y^2,\ V_1 = \frac{\delta^2}{3}\left(H_1 y^3 - 2H_1^3 y - \frac{1}{4} y^4\right), \tag{1}$$

and the thickness of the film is $H_1(x) = H_1(0)\exp(-2x/3)$.

The linear stability of a steady axisymmetric flow is studied using the assumption of local plane-parallelity, when the ratio of the characteristic scale along the x coordinate under perturbation to the corresponding scale for the fundamental flow is assumed to be a small quantity. The non-steady-state solution of the problem is represented in the form

$$\begin{gathered} u(x,\ \theta,\ y,\ s) = U_1(x_0,\ y) + H_0^2 u_1(\xi,\ \theta,\ \eta,\ \tau),\ \ v = V_1 + H_0^2 v_1, \\ w = H_0^2 w_1/\varepsilon_0,\ \ p = H_0 p_1/\varepsilon_0,\ \ h = H_1 + H_0 h_1, \end{gathered} \tag{2}$$

where $H_0 = H_1(x_0)$; $\varepsilon_0 = \varepsilon(x_0)$; $\xi = (x - x_0)/(\varepsilon_0 H_0)$; $\eta = y/H_0$; $\tau = s/H_0^2$. With accuracy up to terms of order ε_0, the transformation of independent variables corresponds to the use of the thickness of the film $H_* = H_C H_0$ for $r = r_0 \equiv R \exp x_0$ as the characteristic length. Substituting the solution (2) into the equations and boundary conditions of the full formulation of the problem and neglecting terms of order ε_0 and δ^4, one can obtain

$$\begin{gathered}
\frac{\partial u_1}{\partial \tau} + \mathrm{Re}\left[(U + u_1)\frac{\partial u_1}{\partial \xi} + \left(U' + \frac{\partial u_1}{\partial \eta}\right) w_1\right] = -\frac{\partial p_1}{\partial \xi} + \frac{\partial^2 u_1}{\partial \xi^2} + \frac{\partial^2 u_1}{\partial \eta^2} - \mathrm{D}^2\left(\frac{\partial u_1}{\partial \theta} - 2v_1\right), \\
\frac{\partial v_1}{\partial \tau} + \mathrm{Re}\left[(U + u_1)\frac{\partial v_1}{\partial \xi} + \left(V' + \frac{\partial v_1}{\partial \eta}\right) w_1\right] = \frac{\partial^2 v_1}{\partial \xi^2} + \frac{\partial^2 v_1}{\partial \eta^2} - \mathrm{D}^2\left(\frac{\partial v_1}{\partial \theta} + 2u_1\right), \\
\frac{\partial w_1}{\partial \tau} + \mathrm{Re}\left[(U + u_1)\frac{\partial w_1}{\partial \xi} + \frac{\partial w_1}{\partial \eta} w_1\right] = -\frac{\partial p_1}{\partial \eta} + \frac{\partial^2 w_1}{\partial \xi^2} + \frac{\partial^2 w_1}{\partial \eta^2} - \mathrm{D}^2 \frac{\partial w_1}{\partial \theta}, \\
\frac{\partial u_1}{\partial \xi} + \frac{\partial w_1}{\partial \eta} = 0, \quad \eta = 0{:}\ u_1 = v_1 = w_1 = 0, \\
\eta = 1 + h_1{:}\ \ \frac{\partial h_1}{\partial \tau} + \mathrm{Re}\left[(U + u_1)\frac{\partial h_1}{\partial \xi} - w_1\right] + \mathrm{D}^2 \frac{\partial h_1}{\partial \theta} = 0, \\
\left[1 - \left(\frac{\partial h_1}{\partial \xi}\right)^2\right]\left(U' + \frac{\partial u_1}{\partial \eta} + \frac{\partial w_1}{\partial \xi}\right) + 2\frac{\partial h_1}{\partial \xi}\left(\frac{\partial w_1}{\partial \eta} - \frac{\partial u_1}{\partial \xi}\right) = 0, \\
V' + \frac{\partial v_1}{\partial \eta} - \frac{\partial h_1}{\partial \xi}\frac{\partial v_1}{\partial \xi} = 0, \\
p_1 - \frac{2}{b}\left[\frac{\partial w_1}{\partial \eta} + \frac{\partial h_1}{\partial \xi}\left(\frac{\partial h_1}{\partial \xi}\frac{\partial u_1}{\partial \xi} - U' - \frac{\partial u_1}{\partial \eta} - \frac{\partial w_1}{\partial \xi}\right)\right] + \frac{\mathrm{Re}}{\mathrm{We}\, b^{3/2}}\frac{\partial^2 h_1}{\partial \xi^2} = 0, \quad b = 1 + \left(\frac{\partial h_1}{\partial \xi}\right)^2,
\end{gathered} \tag{3}$$

where $\mathrm{Re} = U_* H_*/\nu$; $\mathrm{We} = \rho U_*^2 H_*/\sigma$; $U_* = r_0 \Omega^2 H_*^2/\nu$; $U = U_1/H_0^2$; $V = V_1/H_0^2$. The prime in expressions (3) denotes differentiation with respect to the variable η, and the velocity components of the fundamental flow depend on x_0 as a parameter.

To investigate the stability of the fundamental solution, the problem (3) is linearized and one considers the wave solutions of the form $f_1(\xi,\ \theta,\ \eta,\ \tau) = f_2(\eta)\exp i(\alpha\xi + n\theta - \omega\tau)$ for which one can obtain, after some transformations [2],

$$\begin{gathered} w^{\mathrm{IV}} - 2\alpha^2 w'' + \alpha^4 w - i\alpha\,\mathrm{Re}\,[E(w'' - \alpha^2 w) - E'' w] - 2i\alpha\,\mathrm{D}^2 v' = 0, \\ v'' - (\alpha^2 + i\alpha\,\mathrm{Re}\,E)\, v - 2i\alpha^{-1}\mathrm{D}^2 w' - \mathrm{Re}\,V' w = 0, \end{gathered} \tag{4}$$

$$\eta = 0{:}\ w = w' = v = 0, \tag{5}$$

$$\begin{gathered} \eta = 1{:}\ w'' + (\alpha^2 - E'' E^{-1})\, w = 0, \quad v' - iV''(\alpha E)^{-1} w = 0, \\ w''' - (3\alpha^2 + i\alpha\,\mathrm{Re}\,E)\, w' + i\alpha^3\,\mathrm{Re}\,(\mathrm{We}\,E)^{-1} w - 2i\alpha\,\mathrm{D}^2 v = 0 \end{gathered} \tag{6}$$

where $E = U + \gamma - c$; $\gamma = n\mathrm{D}^2(\alpha\mathrm{Re})^{-1}$; $c = \omega(\alpha\mathrm{Re})^{-1}$; and the subscript of the amplitude functions v_2 and w_2 has been omitted.

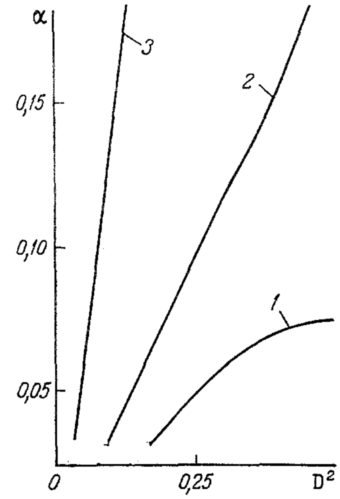

Fig. 1

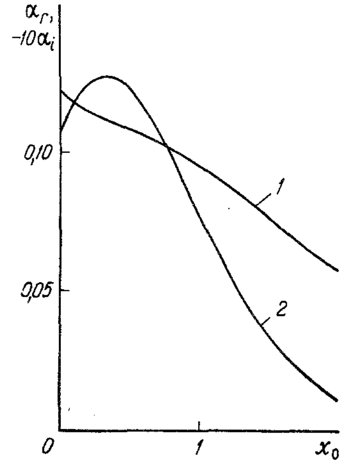

Fig. 2

Fig. 1. Curves of neutral stability of concentric perturbations with space periodicity for the fundamental flow with velocity profile (1) for F = 0.0001: 1) G = 50; 2) 100; 3) 400.

Fig. 2. Radial dependence of the stability characteristic for the most intensely growing perturbations with space periodicity for the fundamental flow with the velocity profile (1) for F = 0.0001 in the case of initial conditions for x = 0, D^2 = 0.5162, G = 111.3: 1) wave numbers α_r; 2) amplification coefficients α_i.

The equations and boundary conditions (4)-(6) make it possible to consider perturbations with space or time periodicity. The determination of these perturbations is associated with the solution of eigenvalue problems in which one calculates the velocity of perturbations Rec or the complex wave number α, respectively. The fundamental solution is unstable if there exist eigenvalues with a positive imaginary part for perturbations with space periodicity, or with a negative imaginary part for perturbations periodic in time.

By way of example of the parameters of the problem (4)-(6), we used the numbers F, G, and D, and $\varepsilon_0 H_0 = D/G$, $Re = GD^3$, $We = FG^2D^5$.

It should be noted that, for films of relatively small thickness ($D^2 \ll 1$), the solution of problem (4)-(6) for concentric (n = 0) perturbations with space periodicity is similar to the solution of the problem of the fundamental flow has the form $U = \eta - \eta^2/2$ [1]. In the case of long perturbations ($\alpha \ll 1$) and intermediate numbers Re($\alpha Re \sim 1$), this problem has an asymptotic solution for which, near the neutral stability curve [1],

$$c_a = 1 + (i - 0.4\,\alpha\,\mathrm{Re})\,\frac{\alpha\,\mathrm{Re}}{9}\,\frac{1 - 3\alpha^2\,\mathrm{We}^{-1}}{1 + 0.16\,(\alpha\,\mathrm{Re})^2}. \tag{7}$$

Problem (4)-(6) is solved by the numerical method of differential driving with orthonormalization of the partial solutions in an internal point [3]. We determined three linearly independent solutions for which the initial conditions for $\eta = 1$ satisfy relations (6). The characteristic equation for the eigenvalues follows from conditions (5) and from the nontriviality of the solution. The numerical algorithm was tested by using solutions (7), for example, for $\alpha = 0.01$, and conditions of Table 1 which gives a number of eigenvalues; the asymptotic solution is then $c_\alpha = 0.9996 + i0.01094$.

It should be noted that, within the adopted model, the study of stability of the fundamental flow for perturbations with periodicity with respect to the angle θ reduces to an analysis of concentric perturbations since the dependence on the parameters for perturbations with space periodicity has the form $c = \gamma + f(\alpha;\ F, G, D;\ U, V)$ and for time periodic perturbations, $\alpha = f(\omega - nD^2;\ F, G, D;\ U, V)$.

The calculations were carried out for the values $0.0001 \leqslant F \leqslant 0.001$, $50 \leqslant G \leqslant 2000$, which are characteristic for the experimental works [2, 4, 5]. We considered films of rela-

TABLE 1. Eigenvalues for Concentric Perturbations with Space Periodicity for the Fundamental Flow with Velocity Profile (1) for F = 0.0001, G = 100, Re = 10

α	c_r	c_i	α	c_r	c_i
0,01	0,9655	0,01048	0,06	0,9563	0,02890
0,02	0,9640	0,01991	0,07	0,9566	0,02032
0,03	0,9619	0,02731	0,08	0,9592	0,006308
0,04	0,9596	0,03178	0,09	0,9651	—0,01352
0,05	0,9575	0,03253			

TABLE 2. Eigenvalues for Concentric Perturbations with Time Periodicity for the Fundamental Flow with Velocity Profile (1) for F = 0.0001, G = 100, Re = 10

ω	α_r	α_i	ω	α_r	α_i
0,1	0,01036	—0,0001164	0,6	0,06279	—0,001784
0,2	0,02673	—0,0004433	0,7	0,07324	—0,001240
0,3	0,03115	—0,0009135	0,8	0,08328	—0,00003910
0,4	0,04164	—0,001412	0,9	0,09262	0,001723
0,5	0,05220	—0,001772			

tively small thickness for which $D^2 \leqslant 0.5$. An example of the calculation is shown in Fig. 1 where the region of instability lies below the corresponding curves.

For the fundamental flow (1) for F = 0.001, the regions of instability for fixed values of the parameter G grow in comparison with the case F = 0.0001. This indicates a stabilizing effect of the surface tension.

By continuing the solution through the curve of neutral stability, one can obtain perturbations which are periodic in time. For these perturbations, Table 2 gives examples of eigenvalues.

Using the hypothesis that the perturbations which are realized in experiments have the largest amplification coefficient, one can calculate the wavelengths for concrete steady flows. Figure 2 shows an example of such calculations.

The results obtained within the adopted model agree with the data [2] where $c_r = 0.57$-0.93. The values of $c_r = 2.7$-4 from [5] are too high. The magnitude of the wave numbers was not compared because the information about the experimental conditions in that work is limited.

NOTATION

Ω, angular velocity of disk rotation; ρ, ν, σ, density, kinematic viscosity, and the surface tension of the liquid; t, time; r, θ, z, stationary cylindrical coordinate system; u_r, u_θ, u_z, velocity components; p_f, pressure; H_c, characteristic thickness of the film; R, smallest radius of the region of the flow; r_0, radius at which the stability is studied; H_*, thickness of the film for $r = r_0$; and $D = H_*\sqrt{\Omega/\nu}$, $G = r_0\sqrt{\Omega/\nu}$, $F = \rho\sqrt{\nu^3\Omega}/\sigma$, parameters of the problem.

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